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# Vladimir Rabinovich: a Mathematician, Colleague and Friend

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Dedicated to the 70th anniversary of Professor Vladimir Rabinovich

It was my pleasure to accept the invitation to write this introductory paper to the volume dedicated to the 70th anniversary of Vladimir Samuilovich Rabinovich, my university colleague in the "previous life" in the Soviet Union, collaborator and friend for more than 40 years.

Vladimir Rabinovich, known for his friends and most of the colleagues as Volodya Rabinovich, was born in Kiev on September 2, 1940, where his childhood passed. In the beginning of the Nazi invasion of USSR in 1941, when many people were evacuated from the Western regions to the interior parts of the country, his



V. Rabinovich at his working  $\underline{\mbox{table}},$  México City, May 2012.

family went from Kiev to the city of Kuibyshev on Volga river (the city of Samara before 1935 and after 1991). Most likely I could not write this article otherwise. They returned to Kiev in 1947.

In 1961 he became a student of the Department of Mechanics and Mathematics of the *Rostov State University* in the Soviet Union. He graduated from this department with Diploma of Honour in 1966. He started his mathematical career at the Chair "Differential and Integral Equations" of the same department as a Ph.D. student in 1966–1969 years, under the guidance of Professor V.A. Kakichev, who noticed Volodya Rabinovich as a capable student and drew him into the world of mathematics. His scientific interests, already during the Ph.D. studies were heavily influenced by the professor of the same Department, well known mathematician I.B. Simonenko. In 1969 Volodya defended Ph.D. Theses and took a position of assistant professor at the same chair, but later moved to the chair "Algebra and discrete mathematics", guided by I.B. Simonenko.

In 1972 he became Associate Professor of the same Department of Mechanics and Mathematics, and Full Professor in 1994.

In 1998 Volodya leaves Russia and moves to Mexico where he took position of the full professor at the *National Polytechnic Institute of Mexico* in Mexico-City, where he continuous to work till present.

Under supervisorship of V. Rabinovich there were defended 8 Ph.D. thesis in Russia, and 3 in Mexico. He is a member of Editorial Boards of various international journals, in particular, "Complex Variables and Elliptic Equations", "Communications in Mathematical Analysis", "Journal of Pseudodifferential Operators", "Mathematics in Engineering, Science and Aerospace".

The first studies of V. Rabinovich were related to the investigation of the Fredholm properties of the multidimensional Wiener-Hopf equations in unbounded domains in  $\mathbb{R}^n$  with the cone type structure at infinity. These results were published in the papers [37] (1967), [38] (1968) and [40] (1969). But the most important results of V.Rabinovich of that time were obtained during the last years of his Ph.D. studies. They were about the Fredholm properties of the general boundary value problems for pseudodifferential operators in such unbounded domains, published in the leading Soviet journals *Mathem. Sbornik* and *Doklady Akademii* Nauk, see [41] (1969) and [44] (1971). These papers were the first ones where the general boundary value problems for pseudodifferential operators were considered in unbounded domains. These results were included in his Ph.D.

Among the most important results obtained by V. Rabinovich in 1972–1977 there were the solvability of the Cauchy and Goursat problems for parabolic pseudodifferential operators, Fredholm properties of pseudodifferential operators and boundary value problems for them on non compact manifolds [46] (1972), [48] (1973), [52] (1975) and [56] (1979).

His paper [51] (1974) devoted to the multi-dimensional convolution operators in the space with exponential weights is worth of special mentioning. In this paper Vladimir Rabinovich



Caucasus, Mount Elbrus region, July 2010.

he extended the well-known results of I. Gohberg and M. Krein to a class of multidimensional convolution operators. It gave a start to his further studies of partial differential operators and pseudodifferential operators in spaces with exponential weights. Thus in the paper [28] (1978) he introduced a class of pseudodifferential operators with analytical symbols in a tube domain in  $\mathbb{C}^n$  and obtained effective results on the boundedness of pseudodifferential operators together with the study of their Fredholm properties in spaces with exponential weights and exponential decreasing of solutions of pseudodifferential equations at infinity.

The series of his papers [60] (1982), [61] (1983) and his papers with a Ph.D. student R. Babadjanian [1] (1985), [2] (1986) and [3] (1987) he studied Fredholm properties of pseudodifferential-difference operators, integral-difference and differential-difference operators. In particular in [2] there was proved the important Theorem on the Wiener-Hopf factorization of the operator-valued functions in the Wiener algebra.

The next important scientific results of the V. Rabinovich are connected with the so-called method of limit operators. The idea of the limit operators historically goes back in fact to a paper of J. Favard of 1927 on the existence of solutions to ordinary differential equations with almost periodic coefficients. These results of Favard were extended to the case of elliptic partial differential equations by E. Muhamadiev in a paper of 1981.

V. Rabinovich in fact turned this approach into a powerful general method, nowadays known as the "method of limit operators" by extending it and giving its wide applications to the investigation of the Fredholm properties of pseudodifferential operators, convolution type operators on  $\mathbb{Z}^n$  and  $\mathbb{R}^n$ , general boundary values problems of the Boutet de Monvel type on manifolds with conical structure at infinity and pseudodifferential operators with shifts, etc, in his paper [61] (1985) and in the series of his papers [22, 24] (1985) and [25] (1986) with the Ph.D. student B. Lange and later in his papers [73] (1992), [76] (1993), [77] (1994), [84] (1998), [87] (1999) and [89, 90] (2001).

These investigations were also elaborated and continued in collaboration with S. Roch (Darmstadt) and B. Silbermann (Chemnitz) in the papers [136] (1998), [137, 138] (2001), [113] (2002), [114–116] (2003), [117] (2004), [124, 125] (2007), [141] (2008) and in his papers [95-97] (2003), [98] (2004).

In 2004 there was published the book [139] by V. Rabinovich, S. Roch and B. Silbermann, where there was presented both the techniques of the method of limit operators and the main results on its applications to various problems of operator theory including convolution type operators, discrete and continuous pseudodifferential operators, singular integral operators on Carleson curves and finite sections method.

V. Rabinovich and S. Roch discovered that the method of limit operators is a strong-tool for the investigation of the essential spectra of the electromagnetic Schrödinger operators on  $\mathbb{R}^n$ ,  $\mathbb{Z}^n$ , and on periodic graphs, which was realized an developed in the papers [98] (2004), [101, 102] (2005), [103, 120] (2006), [123] (2007) [127–129, 141] (2008) [130–132] (2009), [134] (2010). In particular, by means of the method of limit operators there was obtained a simple and short proof of the well-known Hunziker-van Winter-Zhislin Theorem on the essential spectra of the multiparticle Schrödinger operators, together with some new results on the essential spectra of the Schrödinger and Dirac operators.

The area of mathematical interests of V. Rabinovich, as may be already seen from the above, includes various topics from both Analysis and Mathematical. In reality it is wider than has been described in the previous lines. We could mention many others. For instance, it is worthwhile mentioning his studies of the exponential decrease of solutions of differential and pseudodifferential equations. In the papers [12] (1997), [99] (2004), [134] (2006), [127, 128] (2008) and [106, 130, 131, 133] (2009) there were obtained strong and exact results on the behavior of solutions of the differential and pseudodifferential equations with increasing discontinuous coefficients, which in particular included a far going generalizations of the well-known S. Agmon results on the exponential decrease of solutions of elliptic second-order partial differential equations. Note that there was also given applications to the study of exponential decrease of eigenfunctions corresponding to the discrete spectra of the electromagnetic Schrödinger and Dirac operators for wide classes of potentials.



London, Conference dedicated to the 80th birthday of M.Z. Solomyak, September 2011.

Special words should be said about his studies of algebras of singular integral operators on a class of composed Carleson curves with coefficients having oscillating discontinuity, which go back to his interests still-from his research at the Department of Mechanics and Mathematics of the Rostov State University and runs-through his life up to the present time. In the papers [71] (1991), [80] (1995) and [81, 82] (1996), there was shown that in the case where curves, coefficients and weights oscillate, the usual Mellin transform, which is the effective tool in the case of Lyapunov curves and piecewise continuous coefficients and non-oscillating weights, should be replaced by the Mellin pseudodifferential operators with variable and non stabilized symbols.

These investigations were continued in the papers with A. Böttcher (Chemnitz) and Yu. Karlovich (Cuernavaca, Mexico) in the papers [4] (1996), [5] (1998), [6] (2000), [7] (2001), where in a transparent form there was explained the appearance of the logarithmic spirals and logarithmic horns in the local spectra of singular integral operators on a class of composed Carleson curves.

Recently he turned to the studies in a new and quickly developing area known as "Variable Exponent Analysis". In the papers [143] (1997), [144] (2008) and [145] (2011) with the author of this article there were studied singular integral operators and also pseudodifferential in variable exponents Lebesgue spaces, including the case of composite Carleson curves. In particular, in [145] (2011) the Simonenko local principle was extended to the case of variable exponent Lebesgue spaces where the main problem as to localize the space itself.

The task to overview all the studies of Volodya Rabinovich is too enormous for this introductory article, but we still mention a few. In the papers [147, 154] (2000), [148, 149] (2001), [150] (2002) and [151] (2004) with B.-W. Schulze and N. Tarkhanov (Potsdam) there were studied Fredholm properties of boundary value problems in domains with cuspidal points and cuspidal edges and also described the behavior of solutions near singular manifolds of the boundary.

In another cycle of papers [29, 30] (2008) and [31] (2009) with Ya. Lutsky (Karmiel, Israel) he investigated the invertibility of the homogeneous Cauchy problem for parabolic pseudodifferential operators with discontinuous and increasing symbols, together with the study of the behavior of solutions at infinity and near the sets of discontinuities of the symbols.

His interests varied from rather pure mathematical topics in Operator Theory and Mathematical Physics to very applied fields, such as acoustic problems, wave propagation etc. In the papers [69] (1990), [14] (1996) and [33] (1998) with his Ph.D. student O. Obrezanova and the colleague S.M. Grudskii there solved some theoretical and applied problems of the underwater sound propagations on the large distances in the ocean. In particular, there were obtained effective asymptotic formulas for the acoustic fields in the ocean generated by non uniformly moving sources. These investigations were continued later after he moved to Mexico, with his Mexican Master and Ph.D. students in the papers [146] (2003), [34] (2005), [35] (2007), [36] (2009), [109] (2010). The reader can also find other topics of Vladimir Rabinovich's interests in the titles of his publications in the end of the article.

<u>Volodya is a happy person having a nice family with his wife Nelli, his two</u> beautiful daughters Katya and Masha live in Israel, he enjoys his status of grandfather and has two granddaughters and a grandson.

He is very sociable and liked by friends as the life and soul of the party. He has an an outward-looking personality which enables him to get along with peoples at different levels and easily contact with colleagues and all the people around. He is also a sportive person. I remember him playing football when a student at the Rostov State University. He was a member of the student football team of the department and also of a combined team of the university and till his move to Mexico played football in professors' teams in the Rostov State University. From his student's studies till these days, every year he spends some time in mountains, his hiking there being on a serious alpinist's level. These days, his friends wish him to keep a good sportive form, and keep in general a keen interest to mathematics, mountains, social life, for many and many years ahead.

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